11th Conference Mathematics in Technical and Natural Sciences

$$\frac{\partial^2 u}{\partial x \partial y}$$

Krynica, 8 – 12.09.2010

Conference Agenda

Wednesday, 8.09.2010

15:30	Departure of the conference bus from Kraków (AGH) to Krynica
20:00	Supper

Thursday, 9.09.2010

8:00	Breakfast
9:00 – 9:10	**Opening of the Conference**
	Chairman: Roman Popovich
9:10 – 10:10	**Robert Kersner**, *Evolution of turbulent domains: asymptotically self-similar solutions*
10:15 – 10:55	**Oleksandr Makarenko**, *New approaches for reducing non-physical oscillations and smoothing in numerical solutions to evolutionary equations*
10:55 – 11:10	Coffee break
11:10 – 11:50	**Henryk Leszczyński**, *Finite difference methods for evolution equations*
11:55 – 12:35	**Marek Danielewski**, *The Planck-Kleinert Crystal*
13:00	Dinner
	Chairman: Henryk Leszczyński
14:30 – 15:10	**Zygmunt Wronicz**, *On some application of chebyshevian splines to piecewise exponential splines of order 4*
15:15 – 15:40	**Tadeusz Jankowski**, *Difference-functional equations*
15:45 – 16:10	**Antoni Augustynowicz**, *Extreme solutions to ordinary differential equations and Darboux problem on the time scale*
16:10 – 16:25	Coffee break
16:25 – 16:50	**Jerzy Stochel**, *Generalized commutation relations*
16:55 – 17:20	**Piotr Zwierkowski**, *Semi-discrete method of characteristics for McKendrick-von Foerster equations*
17:25 – 17:50	**Lucjan Sapa**, *Explicit and implicit methods for differential functional parabolic equations*
19:00	Supper

Friday, 10.09.2010

8:00	Breakfast
	Chairman: Vsevolod Vladimirov
9:00 – 9:40	**Roman Popovich**, *Local and potential conservation laws of differential equations*
9:45 – 10:25	**Maxim Pavlov**, *Integrability of the nonlocal Kinetic equation for a soliton gas*
10:25 – 10:40	Coffee break
10:40 – 11:20	**Ivan Tsyfra**, *Non-point symmetry and reduction of differential equations*
11:25 – 11:50	**Cristina Sardon**, *Non Isospectral hierarchies arising from a Camassa-Holm hierarchy in* $2+1$ *dimensions (Co-author: P. G. Estevez)*
11:55 – 12:20	**Petr Vojcak**, *Recursion operator, Hamiltionian and symplectic structure for the Mikhailov-Novikov-Wang system*
12:25	Dinner
13:15	Trip
20:00	Supper

Saturday, 11.09.2010

8:30	Breakfast
	Chairman: Robert Kersner
9:30 – 10:30	**Vsevolod Vladimirov**, *Travelling Wave Solutions Supported by the Generalized Burgers Equation*
10:35 – 11:00	**Sergii Skurativskyi**, *Travelling wave solutions of nonlocal models for media with oscillating inclusions*
11:05 – 11:30	**Małgorzata Zdanowicz**, *A mixed problem for quasi-linear hyperbolic system with coefficients, functionally dependent on solutions (Co-author: Z. Peradzyński)*
11:30 – 11:45	Coffee break
11:45 – 12:10	**Michał Nowak**, *The smoothness of solutions to perturbed convolution equations (Co-author: P. Cojuhari)*
12:15 – 12:40	**Maria Malejki**, *Approximation and asymptotic behaviour of eigenvalues for a class of block Jacobi matrices*
13:00	Dinner
	Chairman: Oleksandr Makarenko, Marek Danielewski
14:30 – 14:55	**Michał Góra**, *Robust stability of fractional polynomials*
15:00 – 15:25	**Agnieszka Kowalik**, *On the structure of the universal coverings of homeomorphism groups (Co-author: T. Rybicki)*
15:30 – 15:55	**Ilona Michalik**, *Generalization of a theorem of Ling (Co-author: T. Rybicki)*
15:55 – 16:10	Coffee break
16:10 – 16:35	**Stanisław Kasprzyk**, *Extending the applicability of the method of separation of variables in discrete-continuous systems (in Polish)*
16:40 – 17:05	**Anna Szafrańska**, *Implicit difference methods for infinite systems of hyperbolic functional differential equations*
19:00	Supper **Closing of the Conference**

Sunday, 12.09.2010

8:30	Breakfast
10:30	Departure of the conference bus from Krynica to Kraków

Abstracts

EXTREME SOLUTIONS TO ORDINARY DIFFERENTIAL EQUATIONS AND DARBOUX PROBLEMS ON THE TIME SCALE

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Equations on the time scale include, in particular, equations with classical derivative and recursive equations.

Let $\mathbf{T} \subset \mathbf{R}$ be a closed set, $t_0 = \inf \mathbf{T} > -\infty$. Function $x^{\Delta}(t)$ is called a Δ-derivative of the function $x : \mathbf{T} \to \mathbf{R}$ at the point $t \in \mathbf{T}$, if for all $\epsilon > 0$ there exists a $\delta > 0$, such that

$$|x(\sigma(t)) - x(s) - x^{\Delta}(t)(\sigma(t) - s)| \le \epsilon|\sigma(t) - s|, \quad s \in \mathbf{T} \cap (t - \delta, t + \delta)$$

where $\sigma(t) = \min \mathbf{T} \cap (t, \infty)$. It follows that $x^{\Delta} = x'$, if $\mathbf{T}$ is an interval, and that x^{Δ} in the discrete case is the difference quotient.

Consider the existence of extreme solutions to the problem

$$x^{\Delta}(t) = f(t, x(t), x(h(t))), \quad x(t_0) = x_0$$

and Darboux problem for the following equation

$$u^{\Delta\Delta}_{xy}(x, y) = F(x, y, u(x, y), u(H(x, y))) \ .$$

It is important that the functions h and H do not need to be delays.

THE PLANCK-KLEINERT CRYSTAL

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The most remarkable feature of the deterministic laws of classical physics used in materials science is the wide range of time, place, and scale on which they hold. This talk will review some recent efforts to understand how quantum theory, gravity and electromagnetism emerge from the domain of classical predictability of every day experience. I review two key developments that form a foundation of the Planck-Kleinert Crystal hypothesis: 1) the bi-velocity method (known also as Darken method) and 2) the volume continuity law.

The Planck-Kleinert Crystal hypothesis will be presented for the ideal cubic fcc crystal formed by Planck particles. In this type of quasi-continuum the energy, momentum, mass and volume transport are described by the classical balance equations. The transverse wave is the electromagnetic wave and its velocity equals the velocity of light. The quasi-stationary collective movement of mass in the crystal is equivalent to the particle (body) and such an approach enables derivation of the Schrödinger equation. The diffusing interstitial Planck particles create a gravity field. The model predicts four different force fields and vast amount of the "dark matter and dark energy" in the crystal lattice. It allows for the self-consistent interpretation of multiscale phenomena.

ON THE ROBUST STABILITY OF FRACTIONAL POLYNOMIALS

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The robust stability of fractional order polynomials is currently a very intensively studied branch of the robust stability theory. The most important works on this topic were published within last two years. When reading these papers, one can draw the conclusion that many very important results on the robust stability of polynomials (of integer order), playing a key role in many applications, do not extend to the *fractional* case. In [2], the authors shows, among other things, that the most famous result of the robust stability theory, the Kaharitonov's theorem, does not hold in the class of fractional order interval polynomials.

The main aim of the speech is presentation of a fractional version of the so-called Edge Theorem.

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DIFFERENCE-FUNCTIONAL EQUATIONS

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The subject of consideration are difference problems with nonlinear boundary value conditions of the form

$$\begin{cases} \triangle y(k-1) &= (\mathcal{F}y)(k), k \in Z[1,T] = \{1,2,\ldots,T\}, \\ g(y(0),y(T)) &= 0, \end{cases}$$

where operator $\mathcal{F}$ has the form

$$(\mathcal{F}y)(k) = f(k, y(k), y(\alpha_1(k)), y(\alpha_2(k)), \ldots, y(\alpha_r(k))).$$

Using the method of monotone iterations, gives sufficient conditions for the existence of solutions to this boundary problem, in the area bounded by the lower and upper solutions. Corresponding difference inequalities are also examined.

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EVOLUTION OF TURBULENT DOMAINS: ASYMPTOTICALLY SELF-SIMILAR SOLUTIONS

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We consider a nonlinear, non–uniformly parabolic reaction-diffusion system containing a real parameter γ. The unknown solution is $(k(x,t), \varepsilon(x,t)$, they represent the turbulent energy density and the dissipation rate of turbulent energy.

The system itself can be derived from Navier-Stokes system, using Reynolds' decomposition, statistical homogeneity and dimensional analysis.

The standard self-similarity Ansatz ($k = t^{-\alpha} f(xt^{-\beta})$ *etc.*) leads to some solutions which are however not source-type: at $t = 0$ they are more singular than the Dirac function.

I shall give a simple method for finding solutions which are source-type, not self-similar but for large time they are close to self-similar ones. The most important cases $\gamma > 3/2$ and $0 < \gamma < 1$ will be discussed in detail.

ON THE STRUCTURE OF THE UNIVERSAL COVERINGS OF HOMEOMORPHISM GROUPS

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It is well known that the compactly supported identity component $\mathcal{H}_c(M)$ of the group of all homeomorphisms on a manifold M is perfect and simple. It is known as well that the identity component $\mathcal{H}(M)$ of the group of all homeomorphisms on an open manifold M being the interior of compact manifold is perfect (non-simple).

In this talk there will be presented theorems stating that the universal coverings $\widetilde{\mathcal{H}_c(M)}$ and $\widetilde{\mathcal{H}(M)}$ of the groups $\mathcal{H}_c(M)$ and $\mathcal{H}(M)$ respectively are perfect. The foliated case and the problem of boundedness of the groups of question are also investigated. The analogous theorems concerning the universal covering of the Hamiltonian group of a symplectic manifold have important interpretations in physics.

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FINITE DIFFERENCE METHODS FOR EVOLUTION EQUATIONS

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We analyze FDM's for evolution equations with functional dependence. Their stability is based on abstract functional analysis methods with elegant conclusions.

NEW APPROACHES FOR REDUCING NON-PHYSICAL OSCILLATIONS AND SMOOTHING IN NUMERICAL SOLUTIONS TO EVOLUTIONARY EQUATIONS

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The multi-scale problems can often be formalized by evolutionary equations with non-smooth solutions or solutions with discontinuities. It is well-known that, the difference schemes for approximate solutions to evolution equations may have some errors within the interval of theoretical accuracy of the schemes. As the most known errors one can mention artificial smoothing of the solution and oscillations in the solutions near the sharp fronts of the solution. A lot of special tools have been proposed to avoid such effects e.g., artificial viscosity, artificial dispersion, anti-diffusion, ENO etc. But the problem is still open, especially in design of special difference schemes. Thus, in this paper, some theoretical considerations on the errors in numerical computation are presented. In particular, some cases, like the extra smoothing of fronts at the origin of artificial oscillations in the solutions are investigated. It has been proved that the smoothing is originated by dissipation in schemes and oscillations by dispersion of schemes, and therefore, some methods for improving numerical solutions to evolution equations are proposed. In the case of linear equations, the tools proposed can increase the order of accuracy. The artificial viscosity and artificial dispersion for difference schemes of gas dynamics are presented as examples.

A new class of tools to improve numerical solutions is proposed, namely "Langoliers", i.e. special difference operators which should be applied at each time step, after running of original difference schemes. The design of 'Langoliers' allows reducing the dissipative and dispersive errors of schemes. The examples are anti-diffusion, anti-dispersion and specially constructed difference schemes. Thus, the 'Langoliers' are the realization of the new idea in accuracy increasing (using the extra time stencil) which is auxiliary to idea of using spatially extended stencil. Different illustrative examples of such tools are considered for gas dynamics equations and wave equation. Moreover, some computations of new multi-scale problems are considered, e.g. hyperbolic modification of Burger's equation and blow-up solutions.

APPROXIMATION AND ASYMPTOTIC BEHAVIOUR OF EIGENVALUES FOR A CLASS OF BLOCK JACOBI MATRICES

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The theory and methods related to tridiagonal matrices are still developed and generalized. In the context of advances and applications, block tridiagonal matrices are very interesting.

We consider a Jacobi operator J in the Hilbert space $l^2 = l^2(\mathbb{N}, \mathbb{C}^p)$ given by the symmetric block tridiagonal matrix

$$J = \begin{pmatrix} D_1 & C_1^* & 0 & \cdots & \cdots \\ C_1 & D_2 & C_2^* & 0 & \ddots \\ 0 & C_2 & D_3 & C_3^* & \ddots \\ \vdots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}, \quad D_n = D_n^*,\, C_n \in M_{p\times p}(\mathbb{C}),\ n \geq 1.$$

Let d_n^{min} and d_n^{max} be the smallest and the largest eigenvalue of D_n respectively. Under the conditions

$$d_n^{max} = \delta_1 n^\alpha (1 + o(1)), \quad d_n^{min} = \delta_2 n^\alpha (1 + o(1)) \quad (n \to \infty),$$

$$\|C_n\| = O(n^\beta) \quad (n \to \infty), \quad \text{and} \quad \alpha > \beta,$$

the operator J has discrete spectrum and is bounded from below. Moreover, the eigenvalues of J are approximated by the eigenvalues of finite submatrices of the matrix representation of J ([1]). We observe that if $r \in (0, 1)$ is suitably chosen and $\gamma > 0$, then

$$\lambda_n(J) = \mu_n + O(n^{-\gamma}),\ \ n \to \infty,$$

where $\lambda_n(J)$ is the $n-$th eigenvalue of J and μ_n is the $n-$th eigenvalue of the truncated matrix J_N for $N \geq r^{-1}pn$.

Under additional conditions, an asymptotic formula for μ_n can be found in terms of the entries of J_N, and asymptotics for the point spectrum of J can be obtain.

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GENERALIZATION OF A THEOREM OF LING

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An important theorem of Ling states that if G is any factorizable non-fixing group of homeomorphisms of a paracompact space then its commutator subgroup $[G, G]$ is perfect. We introduce further studies on the algebraic structure (e.g. uniform perfectness, uniform simplicity) of $[G, G]$ and $[\tilde{G}, \tilde{G}]$, where $\tilde{G}$ is the universal covering group of G. In particular, we prove that if G is bounded factorizable non-fixing group of homeomorphisms then $[G, G]$ is uniformly perfect.

THE SMOOTHNESS OF SOLUTIONS TO PERTURBED CONVOLUTION EQUATIONS

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In this talk there will be presented analysis of the order of smoothness of solutions to certain classes of perturbed Wiener-Hopf equations, while smoothness is understood as belonging to the respective weight space.

INTEGRABILITY OF THE NONLOCAL KINETIC EQUATION FOR A SOLITON GAS

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We introduce and study a new class of nonlocal kinetic equations, which arise in the description of nonequilibrium macroscopic dynamics of soliton gases with elastic collisions. These equations represent nonlinear integro-differential systems and have a novel structure, which we investigate by studying in detail the class of N -component "cold-gas" hydrodynamic reductions. We prove that these reductions represent integrable linearly degenerate hydrodynamic type systems for arbitrary N which is a strong indication to integrability of the full kinetic equation. We derive compact explicit representations for the Riemann invariants and characteristic velocities of the hydrodynamic reductions in terms of the "cold-gas" component densities and construct a number of exact solutions having special properties (quasi-periodic, self-similar). Hydrodynamic symmetries are then derived and investigated.

The obtained results shed the light on the structure of a large class of integrable hydrodynamic type systems in the continuum limit and are also relevant to the description of turbulent motion in conservative compressible flows.

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LOCAL and POTENTIAL CONSERVATION LAWS of DIFFERENTIAL EQUATIONS

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The basic notions and statements of local and potential conservation laws of differential equations are reviewed. The main attention is paid to constructive methods of finding conservation laws for general systems of differential equations. Equivalence of conservation laws with respect to Lie symmetry groups for fixed systems of differential equations and with respect to equivalence groups or sets of admissible transformations for classes of such systems are considered.

It is proved that potential conservation laws have characteristics depending only on local variables if and only if they are induced by local conservation laws. Therefore, characteristics of purely potential conservation laws have to essentially depend on potential variables. The problem on classification of potential conservation laws is rigorously posed.

The general theory is illustrated by the consideration of conservation laws for evolution equations in two independent variables. In particular, we present normal forms for the equations admitting one or two low-order local conservation laws. Examples include Harry Dym equation, Korteweg–de Vries-type equations, and Schwarzian KdV equation. The possible dimensions of spaces of conservation laws for second-order evolution equations prove to be 0, 1, 2 and ∞, and equations corresponding to the last case are linearizable by contact transformations.

For any linear $(1 + 1)$-dimensional evolution equation of even order its space of conservation laws is exhausted by linear ones and is isomorphic to the solution space of the corresponding adjoint equation. Any similar equation of odd order can additionally have at most quadratic conservation laws. It appears that any potential conservation law of a linear second-order evolution equation is induced by a local one.

The classification of potential conservation laws of diffusion–convection equations with respect to the associated equivalence group and an exhaustive list of locally inequivalent potential systems corresponding to these equations are given as an example on calculation of a complete hierarchy of potential conservation laws.

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EXPLICIT AND IMPLICIT DIFFERENCE METHODS FOR DIFFERENTIAL FUNCTIONAL PARABOLIC EQUATIONS

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Assume that, $T > 0$, $X = (X_1, ..., X_n)$, $\tau_0 \geq 0$, $\tau = (\tau_1, ..., \tau_n)$, where $X_i > 0$, $\tau_i \geq 0$ for $i = 1, ..., n$. Define

$$\begin{aligned} E &= [0, T] \times (-X, X) \subset \mathbf{R}^{1+n}, \\ E_0 &= [-\tau_0, 0] \times [-X - \tau, X + \tau] \subset \mathbf{R}^{1+n}, \\ \partial_0 E &= [0, T] \times ([-X - \tau, X + \tau] \setminus (-X, X)) \subset \mathbf{R}^{1+n}, \\ \Omega &= E \cup E_0 \cup \partial_0 E. \end{aligned}$$

Define also $\Delta = E \times C(\Omega, \mathbf{R}) \times \mathbf{R}^n \times M_{n\times n}$, where $M_{n\times n}$ is the set of all $n \times n$ symmetric real matrices.

Let $f : \Delta \to \mathbf{R}$ and $\varphi : E_0 \cup \partial_0 E \to \mathbf{R}$ be given functions. Consider a nonlinear second-order partial differential functional equation of parabolic type of the form

$$\partial_t z(t, x) = f(t, x, z, \partial_x z(t, x), \partial_{xx} z(t, x)) \tag{1}$$

with the *initial condition* and the *boundary condition of the Dirichlet type*

$$z(t, x) = \varphi(t, x) \text{ on } E_0 \cup \partial_0 E, \tag{2}$$

where $\partial_x z = (\partial_{x_1} z, ..., \partial_{x_n} z)$, $\partial_{xx} z = \left[\partial_{x_i x_j} z\right]_{i,j=1}^n$. Our aim is to give explicit and implicit convergent and stable difference methods for the initial boundary problem (1), (2).

The functional dependence is of the Volterra type (e.g., delays or Volterra type integrals). The equation may be nonlinear with respect to second derivatives. Such an equation is called strongly nonlinear. Nonlinear estimates of the generalized Perron type for f introduced by the author are assumed. Under these generalizations, equation (1) includes, as special cases, a quasi-linear equation and a strongly nonlinear equation with a quasi-linear term ([4], [7]).

If f is a polynomial with respect to the functional variable, then the global generalized Perron type estimates are not fulfilled. But we give the theorems on the estimate of solutions for the differential functional problem (1), (2) and a family of associated difference functional schemes. By these theorems, the numerical methods may be treated in a subspace $C(\Omega, R) \subset C(\Omega, \mathbf{R})$, where we consider the local generalized Perron type conditions ([2]). Hence equation (1) covers equations with the polynomial right-hand sides, e.g., the Fisher equation, the porous media equation, the Newell-Whitehead equation, the Zeldovich equation, the KPP equation, the Nagumo equation, the Huxley equation and others considered in [1], [3].

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NON ISOSPECTRAL HIERARCHIES ARISING FROM A CAMASSA-HOLM HIERARCHY IN $2+1$ DIMENSIONS

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The non isospectral problem (Lax Pair) associated with a hierarchy in $2+1$ dimensions that generalizes the well known Camassa-Holm hierarchy is presented. Here, we have investigated the non classical Lie symmetries of this Lax Pair when the spectral parameter is considered a field. These symmetries can be written in terms of five arbitrary constants and three arbitrary functions. Different similarity reductions associated with these symmetries have been derived. Of particular interest are the reduced hierarchies whose $1+1$ Lax Pair is also non isospectral.

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TRAVELLING WAVE SOLUTIONS OF NONLOCAL MODELS FOR MEDIA WITH OSCILLATING INCLUSIONS

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Complicated reaction of real media to external impacts is connected with the hierarchical internal structure of media, with dynamical processes taking place on the level of structural elements. One of the ways to take into account the features of media structure is the including of additional volumetric forces in the equation of motion. Within the framework of this approach the mathematical model for structured media is considered [1]

$$\rho u_{tt} = \sigma_x - m\rho w_{tt}, \quad w_{tt} + \omega^2 (w-u) = 0,$$
$$\sigma = E_1\varepsilon + E_3\varepsilon^3 + \theta\left(\sigma_{xx} - \sigma_x \frac{\varepsilon_x}{\varepsilon+1} - \eta\left[\varepsilon_{xx} - \frac{1}{\varepsilon+1}\,\varepsilon_x^2\right]\right), \quad \varepsilon = u_x, \tag{3}$$

where ρ and u are the density and the displacement of the main medium, $m\rho$ is the density of oscillating inclusions distributed in the main medium, w is the displacement of a partial oscillator with the natural frequency ω, and $\sigma = \sigma(\varepsilon)$ is the equation of state taking into account the spatially nonlocal effects at the nonzero parameter θ. Solutions of the form $u = U(s)$, $w = W(s)$, $s = x - Dt$ are considered in details. The previous investigations [2] dealing with the model (3) at $\theta = 0$ have shown that the local counterpart possesses the localized wave regimes. The bell-shaped solitary wave solutions were found, and non-analytical solutions with finite support (compactons) were derived as well. Taking into account the nonlocal effects in model (3) makes more complex the phase space of the dynamical system describing the travelling wave solutions. It is shown that homoclinic curves corresponding to solitary waves undergo deformations. Using the Poincare sections technique, at the parameter θ increasing the chaotic regime development is observed in the phase space of the dynamical system. When the deviation from steady state is small, the investigations of model (3) are complemented by analytical treatments. In this case the asymptotic solution is derived by the nonlinear normal modes method. Applying this method, the original dynamical system is transformed into two separated nonlinear subsystems of the lower order. It turns out that the subsystems are integrable. Analytical and qualitative treatments of these systems display the localized regimes. In particular, the heteroclinic trajectories playing the key role in the chaotic regimes production are observed, and the conditions of the heteroclinic contours formation are obtained.

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GENERALIZED COMMUTATION RELATIONS

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Each family of subnormal operators in Hilbert space fulfils the Hamos-Bram condition on a sufficiently dense subset of its domain. It is shown that the generalized commutation relation implies the Hamos-Bram condition for an infinite family of operators. As an example of application of this property it is proved that every family of generalized creation operators in Bargmann space of an infinite order, indexed by mutually orthogonal vectors from l_2, is subnormal. It is also shown that for some classes of operators, including bounded ones and quasi-shifts, the generalized commutation relation implies subnormality.

IMPLICIT DIFFERENCE METHODS FOR INFINITE SYSTEMS OF HYPERBOLIC FUNCTIONAL DIFFERENTIAL EQUATIONS

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The paper deals with classical solutions of initial boundary value problems for infinite systems of non-linear differential functional equations. Two types of difference schemes are constructed. First we show that solutions of our differential problem can be approximated by solutions of infinite difference functional schemes. In the second part of the paper we prove that solutions of finite difference systems approximate the solutions of our differential problem.

We give a complete convergence analysis for both types of difference methods. We adopt nonlinear estimates of the Perron type for given functions with respect to the functional variable. The proof of the stability is based on the comparison technique. Numerical examples are presented.

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NON-POINT SYMMETRY AND REDUCTION OF DIFFERENTIAL EQUATIONS

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We study the symmetry and reduction of nonlinear evolution and wave type differential equations by using operators of non-point symmetry. In our approach we consider operators of classical as well as conditional symmetry. We construct the corresponding ansatz for derivatives of dependent variable which reduces the scalar equation to a system of ordinary differential equations. It appears that the combination of non-point and conditional symmetry allows us to construct not only solutions but also Backlund transformations for the equation under consideration. We also use the operators of Lie-Backlund symmetry for the reduction of partial differential equations which are not restricted to the ones of evolution type.

TRAVELLING WAVE SOLUTIONS SUPPORTED BY THE GENERALIZED BURGERS EQUATION

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In this talk there will be presented the results of investigations of travelling wave (TW) solutions to the hyperbolic generalization of convection-reaction-diffusion equation.

At first, we will present the source equation and outline its connections with the classical transport equations. Next, we will shed light on the ways of obtaining analytical TW solutions, and present some of them. We will also present the TW solutions, which have not been described analytically yet, but have been discovered thanks to qualitative analysis and numerical simulation. In the last part we will tell about stability properties of TW solutions and some unsolved problems.

RECURSION OPERATOR, HAMILTONIAN AND SYMPLECTIC STRUCTURE FOR THE MIKHAILOV-NOVIKOV-WANG SYSTEM

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We study a new two-component fifth-order integrable system

$$u_t = -\frac{5}{3}u_5 - 10vv_3 - 15v_1v_2 + 10uu_3 + 25u_1u_2 - 6v^2v_1 + 6v^2u_1 + 12uvv_1 - 12u^2u_1,$$

$$v_t = 15v_5 + 30v_1v_2 - 30v_3u - 45v_2u_1 - 35v_1u_2 - 10vu_3 - 6v^2v_1 + 6v_u^21 + 12u_v^21 + 12vuu_1, \qquad (4)$$

where $u_i = \frac{\partial^i u}{\partial x^i}$, $\frac{\partial^j v}{\partial x^j}$, which was recently found by Mikhailov, Novikov and Wang [2, 3]. Note that upon setting $v \equiv 0$ system (4) reduces to the well-known Kaup-Kupershmidt equation [1]. Using the so-called symbolic method Mikhailov et al. proved that the system (4) possesses infinitely many generalized symmetries of orders $m \equiv 1, 5 \mod 6$. However, recursion operator and symplectic and Hamiltonian structures for (4) were not known so far. In this talk we present these quantities and explore their properties.

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0N SOME APPLICATION OF CHEBYSHEVIAN SPLINES TO PIECEWISE EXPONENTIAL SPLINES OF ORDER 4

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The purpose of the lecture is to show that piecewise exponential splines may be presented as Chebyshevian splines with some exponential weight functions. Cubic splines are used in many applications because of their smoothness, low computational complexity and their energy minimising property, but the interpolatory cubic spline can exhibit undesirable oscilations between the interpolation points. We may eliminated this fact using piecewise exponential splines. In [1] were constructed interpolating piecewise exponential splines. The authors assumed that the splines are of class C^2. We show that we do not need to assume that the spline ought to be of class C^2. Instead of this we give another condition of regularity. We prove existence and uniqueness of such interpolants and obtain error bounds for approximation by these splines.

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A MIXED PROBLEM FOR QUASI-LINEAR HYPERBOLIC SYSTEM WITH COEFFICIENTS FUNCTIONALLY DEPENDENT ON SOLUTIONS

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A mixed problem for quasi-linear hyperbolic system, with the coefficients, functionally dependent on solutions, is considered. We assume that the coefficients are continuous, nonlinear operators in $C^1(\mathbb{R})$ Banach space which satisfy some additional assumptions. We prove the existence and uniqueness of local solution in time of class $C^1(\mathbb{R})$, assuming that the initial data are also of $C^1(\mathbb{R})$.

SEMI-DISCRETE METHOD OF CHARACTERISTICS FOR McKENDRICK-VON-FOERSTER EQUATIONS

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Suppose that $E = [0,T] \times \mathbf{R}_+$, $\mathbf{R}_+ = [0,+\infty)$, $T > 0$. For given functions $c\colon E \times \mathbf{R}_+ \to \mathbf{R}$, $\lambda\colon E \times \mathbf{R}_+^2 \to \mathbf{R}$, and $v\colon \mathbf{R}_+ \to \mathbf{R}_+$ consider the equation

$$\partial_t u(t,x) + c\,(t,x,z(t))\,\partial_x u(t,x) = u(t,x)\lambda(t,x,u(t,x),z(t)) \tag{5}$$

with the initial condition

$$u(0,x) = v(x), \quad x \in \mathbf{R}_+, \quad \text{where} \quad z(t) = \int_0^\infty u(t,x)\,dx, \quad t \in [0,T]. \tag{6}$$

The condition $c(t,0,q) = 0$, $t \in [0,T]$, $q \in \mathbf{R}_+$, implies that the problem is properly possed. Problem (5)–(6) is transformed into system of ordinary differential equations applying method of characteristics.

Suppose that $h > 0$ is a discretization parameter and there are $C_0, C_1 > 0$ such that $\omega^{(0)} = 0$, $C_0 h \leq \omega^{(j+1)} - \omega^{(j)} \leq C_1 h$ for $0 \leq j \leq N_h - 1$, where N_h is a sufficiently large natural number. By $E_{0.h} = \{\omega^{(j)}, 0 \leq j \leq N_h\}$ we denote the set of the nodal points on some bounded part of the initial set E_0. Consider semi-discrete method of characteristics

$$\frac{d}{ds}\eta^{(j)}(s) = c(s,\eta^{(j)}(s),\zeta(s)), \quad \eta^{(j)}(0) = \omega^{(j)}, \quad \omega^{(j)} \in E_{0.h},$$

$$\frac{d}{ds}\psi^{(j)}(s) = \psi^{(j)}(s)\lambda\left(s,\eta^{(j)}(s),\psi^{(j)}(s),\zeta(s)\right), \quad \psi^{(j)}(0) = v(\omega^{(j)}), \quad 0 \leq j \leq N_h,$$

where

$$\zeta(s) = \sum_{j=1}^{N_h} \psi^{(j)}(s)\left(\eta^{(j)}(s) - \eta^{(j-1)}(s)\right).$$

We present results on boundedness of solutions of our method in supremum and l^1 norms. Theorem on convergence of the method and some illustrative numerical examples will be given.

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